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GROUP ORTHOGONAL ARRAYS FOR ELIMINATION OF MULTIPLE-TIME-AROUND ECHOES IN RADARS

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The advantage of interleaved group-orthogonal/group-complementary arrays lies in their property of sidelobe cancellation over multiple range intervals. This results in reduced clutter interference when tracking low-speed targets in clutter. With this waveform, a high PRF can be used to increase energy on target or increase doppler coverage, and allows detection and tracking of targets in multiple-time-around range intervals with cancellation of clutter responses from the principal unambiguous range interval and from other multiple-time-around range intervals.

Synthesis procedures of arrays to be interleaved to achieve zero responses in multiple-time-around range intervals are presented in this report, along with examples of the zero response features. References are also given to group-complementary coding and other synthesis techniques of orthogonal binary arrays.

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I. BACKGROUND

The unique application of group-complementary and group-orthogonal arrays to radar waveform design allows sidelobe control not only in the principal unambiguous range interval, but also in multiple-time-around range intervals. Previous waveform designs concentrated on controlling responses in the principal unambiguous range interval, and usually, responses for multiple-time-around range intervals were identical to those of the first-time-around range interval. Interleaved group-orthogonal/group-complementary arrays provide sidelobe cancellation in the principal unambiguous range interval as well as for complete sidelobe cancellation in one or more multiple-time-around range intervals. The uniqueness of this waveform design, compared to previous designs, is that it simultaneously controls near-in sidelobes as well as far-out sidelobes in multiple-time-around intervals. Also, a unique aspect of this design is that it allows for placing the maximum response in a multiple-time-around interval while providing cancellation of responses from nearer-in range intervals, including the principal unambiguous range interval.

The advantage of interleaved group-orthogonal/group-complementary arrays lies in their property of sidelobe cancellation over multiple range intervals. This results in reduced clutter interference when tracking low speed targets in clutter. With this waveform, a high pulse repetition frequency (PRF) can be used to increase energy on target or increase doppler coverage, which allows detection and tracking of targets in multiple-time-around range intervals with cancellation of clutter responses from the principal unambiguous range interval and from other multiple-time-around range intervals.

II. INTRODUCTION

Group-complementary arrays can be applied to synthesize a radar waveform with the desirable property of temporal sidelobe cancellation [1]. Figure 1 illustrates the cross-correlation process in the receiver. The group-complementary array included in the figure is composed of rows of shifted maximum-length sequences, with a final "all 1" row and column. Each row biphase codes a single pulse, and the receiver summation is over K pulses corresponding to the K rows of the code array.

The algebraic property which results in zero-valued temporal sidelobes holds in the maximum unambiguous range interval of the equivalent uncoded pulse sequence, $\pm (T-T_1)$, where T is the pulse repetition interval and T_1 is the pulse width. Figure 2 is a timing diagram of the coded pulse sequence for the periodic and aperiodic cases, where each pulse is numbered corresponding to the row from the code array which biphase modulates that pulse.

Figure 3 gives the transmit/reference waveforms aperiodic correlation functions for the group complementary array from Figure 1. Temporal sidelobes are seen to cancel over the temporal range interval of $\pm(T-T_1)$, but sidelobes do exist at multiple-time-around range intervals centered around $\pm T$, $\pm 2T, \ldots,$ $\pm(K-1)T$.

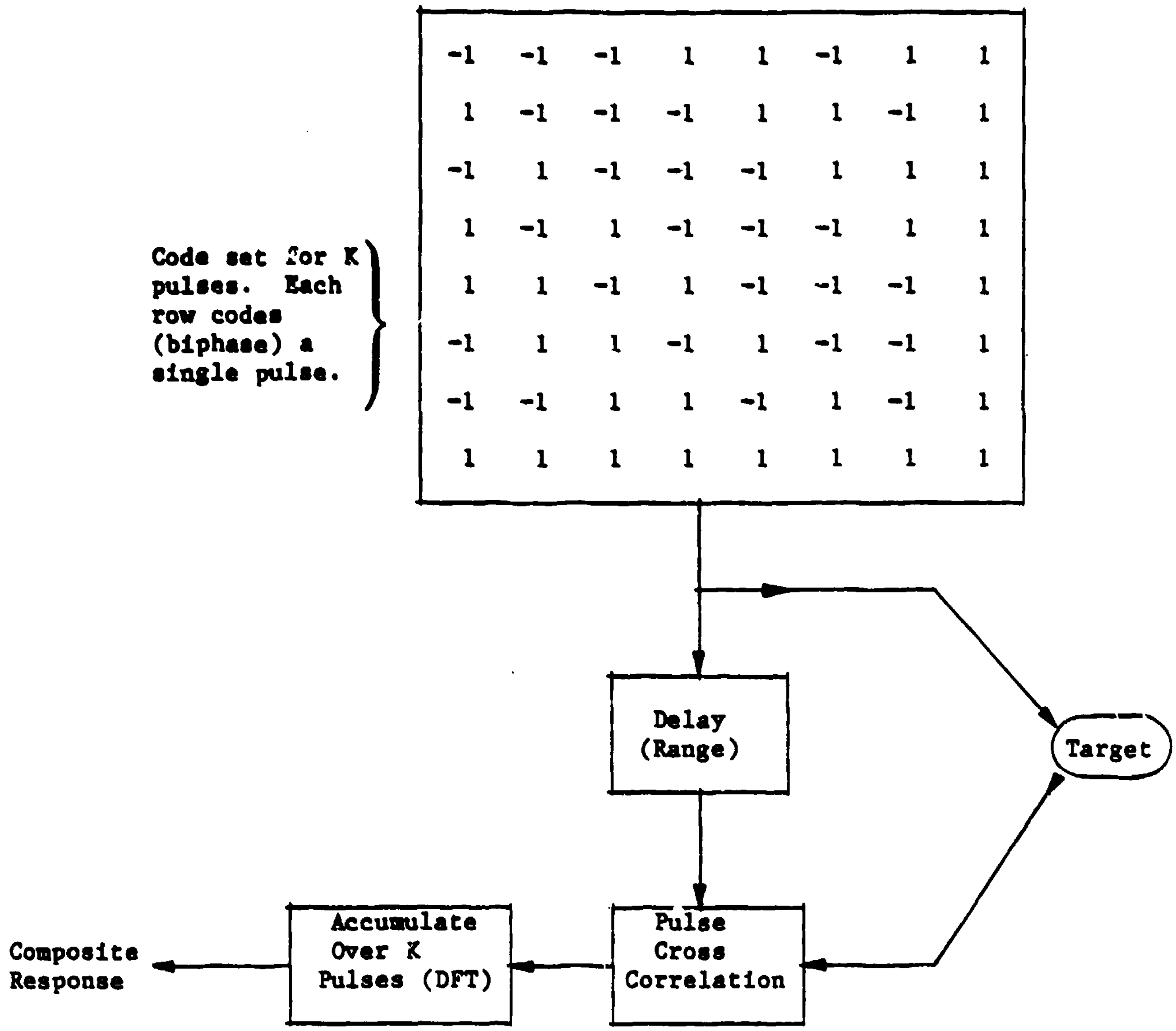
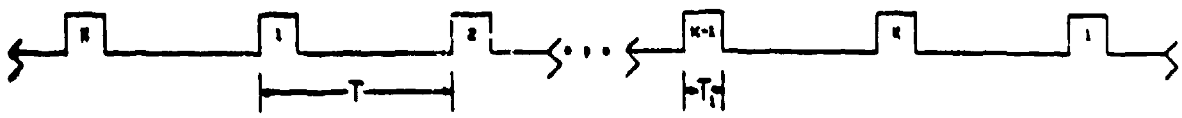
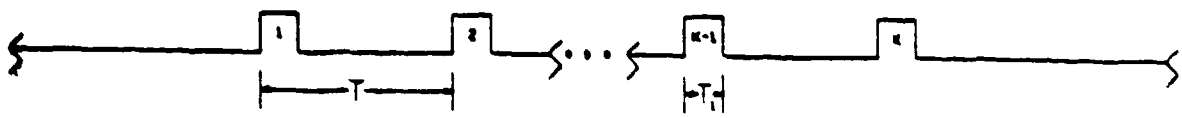


Figure 1. Schematic of radar processing with group-complementary codes.



(a) Periodic case: repeating sequence of K pulses



(b) Aperiodic case: nonrepeating sequence of K pulses

Note: Labels indicate row number from group complementary array.

Figure 2. Pulse sequence timing diagram.

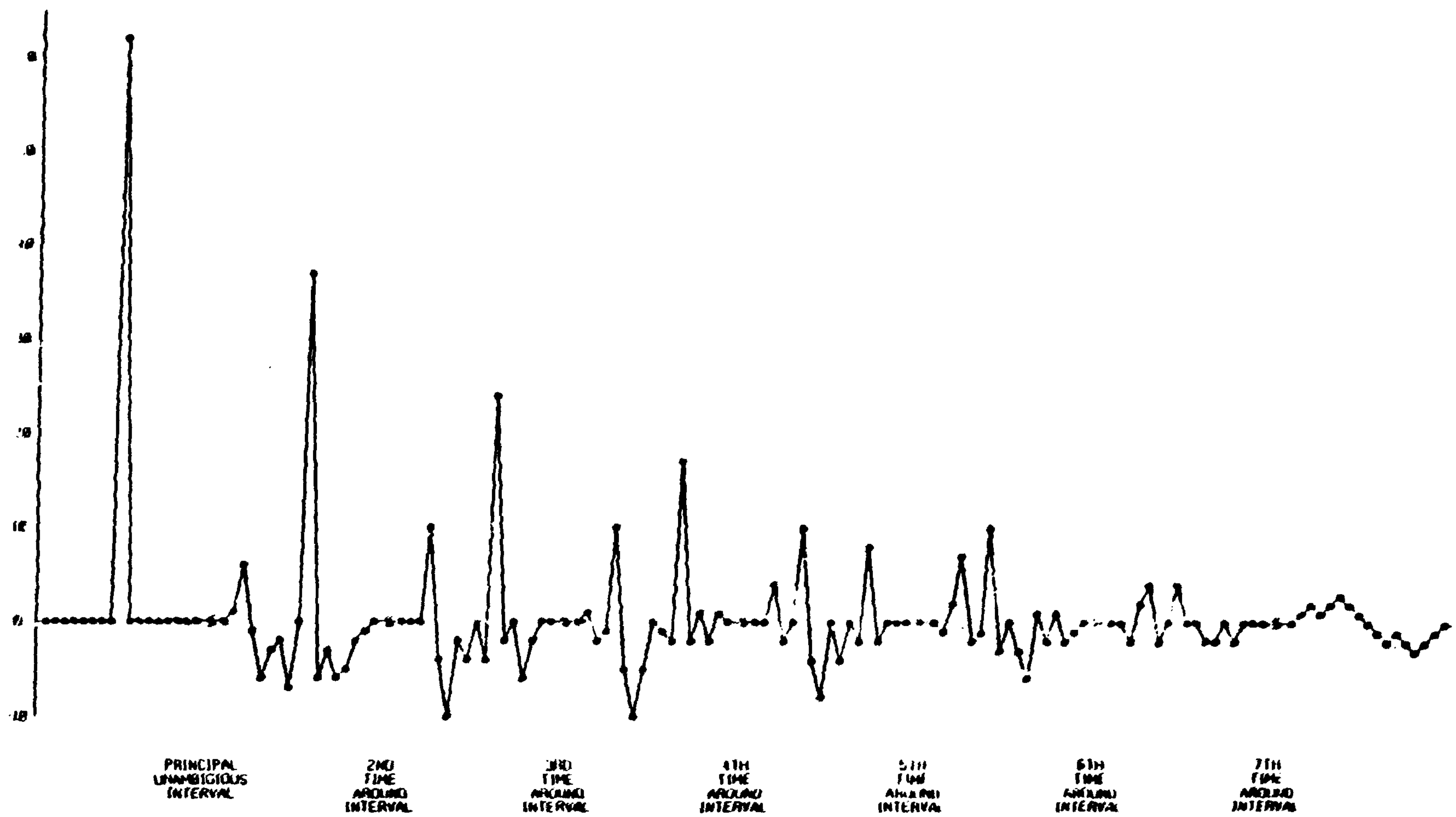


Figure 3. Transmit/reference waveforms - aperiodic cross-correlation, Example 1.

The rows of a group-complementary array can be interchanged without affecting the temporal sidelobe cancellation property in the interval $\pm(T-T_1)$. The array

$$\begin{array}{cccccccc}
-1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\
-1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
-1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & 1
\end{array}$$

results from interchanging several rows of the array in Figure 1. Figure 4 gives the aperiodic correlation functions for this array. The peak sidelobes are seen to be smaller than for the previous example; however, they are still relatively large.

In many cases, it is beneficial to eliminate the temporal sidelobes from multiple-time-around range intervals. It will be shown that the group-complementary array's property of temporal sidelobe cancellation in the $\pm(T-T_1)$ interval can be extended to multiple-time-around intervals through an array synthesis procedure which utilizes a "group-orthogonality" property of certain sets of group-complementary arrays. This extension applies to both the periodic and aperiodic cross-correlation functions.

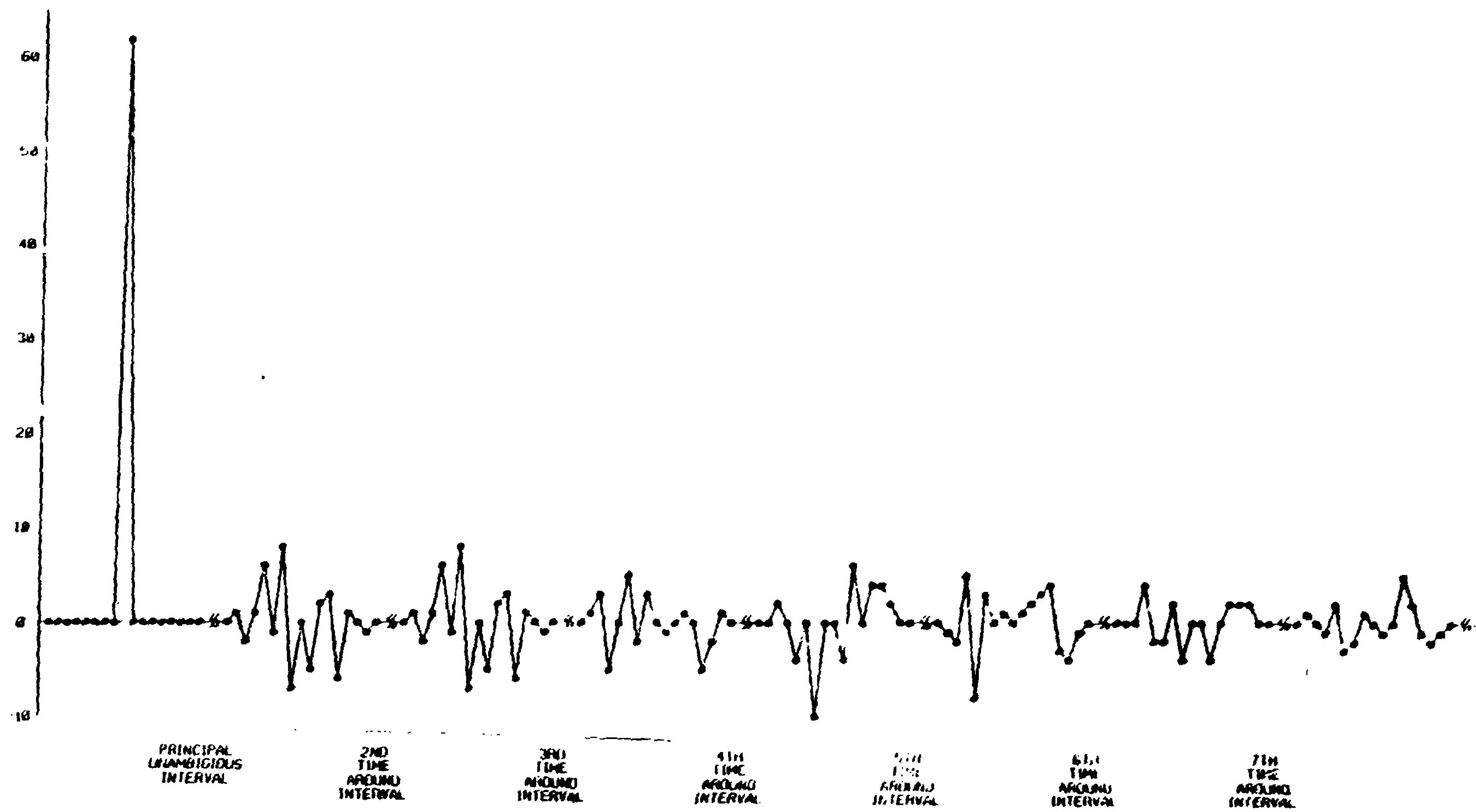


Figure 4. Transmit/Reference waveforms - aperiodic cross-correlation, Example 2.

III. GROUP-ORTHOGONAL ARRAYS

A single group-complementary array can be operated upon to generate a set of group-complementary arrays which have a group-orthogonality property. That is, each array in the set is orthogonal to all other arrays in the set over a $\pm(T-T_1)$ interval of the cross-correlation function of the two pulse sequences which are coded by any two arrays from the set.

The method of generating a group-orthogonal array, A_z, from a group-complementary array, A_0, is to perform a matrix multiplication

$$A_z = A_0 V_z \tag{1}$$

where A_0 is a K row by N column array, and V_z is an N by N diagonal array of the form

$$V_z = \begin{matrix} v_z(1) & 0 & 0 & 0 & \cdots & 0 \\ 0 & v_z(2) & 0 & 0 & \cdots & 0 \\ 0 & 0 & v_z(3) & \cdots & & 0 \\ \cdot & \cdot & \cdot & 0 & \cdots & \cdot \\ 0 & 0 & 0 & 0 & \cdots & v_z(N) \end{matrix} \quad . \tag{2}$$

Even though this report considers only the case where N = K, (with K being a power of 2, group-orthogonal arrays exist for any even N. For $N = K = 2^N$, the diagonal elements of V_z will be elements of Walsh vectors. Each Walsh vector (index z) used in V_z to form A_z results in a new array which is group-orthogonal to A_0. Also, the arrays generated from a set of Walsh vectors will be group-orthogonal to each other.

As an example, consider the group-complementary array

$$A_0 = \begin{matrix} -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix} \tag{3}$$

A set of group-complementary arrays which are also group-orthogonal can be generated from the Walsh vectors, W_z,

$$W_1 = 1 \quad 1 \quad -1 \quad -1, \tag{4}$$

$$W_2 = 1 \quad -1 \quad -1 \quad 1, \tag{5}$$

and

$$W_3 = 1 \quad -1 \quad 1 \quad -1. \tag{6}$$

Then

$$V_1 = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1, \end{matrix} \tag{7}$$

$$V_2 = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1, \end{matrix} \tag{8}$$

and

$$V_3 = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix} \tag{9}$$

The resulting group orthogonal arrays are

$$A_1 = \begin{matrix} -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1. \end{matrix} \tag{10}$$

$$A_2 = \begin{matrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{matrix} \tag{11}$$

and

$$A_3 = \begin{matrix} -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1\ . \end{matrix} \tag{12}$$

The four arrays, A_0, A_1, A_2, and A_3, form a group-orthogonal set of arrays. Figure 5 is the cross-correlation function for A_0 and A_3, and illustrates the orthogonality of the two arrays over the interval $\pm(T-T_1)$.

For $N = 2^n$, the number of arrays in a group-orthogonal set, S, is

$$S = N, \tag{13}$$

corresponding to the N Walsh vectors which can be applied in (2).

The group-orthogonal, array-set synthesis procedure given utilizes group-complementary arrays in the form of Hadamard matrices with the sign pattern of a Walsh vector operating on columns of the original array. Other orthogonal and complementary array synthesis methods are discussed in Reference 2, 3, and 4.

IV. CANCELLATION OF MULTIPLE-TIME-AROUND RESPONSES

A group-orthogonal set of group-complementary arrays can be used to synthesize composite transmitted and reference code arrays. The cross-correlation of these two resulting arrays will have the property of temporal sidelobe cancellation over the maximum unambiguous range interval, and over one or more multiple-time-around range intervals.

The synthesis procedure for the composite code arrays is to interleave rows from the arrays of the group-orthogonal sets to biphase modulate K bursts of pulses, with up to S pulses in each burst. The K bursts will be separated by a time interval equal to one or more pulse-repetition intervals, depending on the number of multiple-time-around intervals which are to have temporal responses cancelled. The resulting K*S pulse sequence can be utilized as the radar waveform on a periodic or aperiodic basis. The composite receiver response is formed from the sum of the K*S pulses.

As an example of this synthesis procedure, the group orthogonal arrays A_0, A_1, A_2, and A_3, given by equations (3), (10), (11), and (12) respectively, will be the basis for the synthesis of a composite waveform. The composite code formed from interleaving rows from A_0, A_1, A_2, and A_3 is

$$
C_T = C_R = \begin{array}{rrrr}
-1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 \\
-1 & 1 & 1 & -1 \\
\\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
\\
-1 & 1 & -1 & 1 \\
-1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 \\
-1 & -1 & -1 & -1 \\
\\
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1
\end{array} . \tag{14}
$$

The timing diagram corresponding to transmitter code, C_T, and reference code, C_R, is shown in Figure 6, and the cross-correlation between C_T and C_R is given in Figure 7. This cross-correlation function corresponds to the zero-doppler cut of the waveform's cross-ambiguity function. As can be seen from the figure, temporal sidelobes have been cancelled over the maximum unambiguous range interval and over the second-time-around range interval of an equivalent, but uncoded, pulse sequence.

Observe that the waveforms shown as C_T and C_R could, by timing of C_R, be used to place the maximum cross-correlation response in the second-time-around range interval (for an uncoded pulse sequence with the same pulse repetition interval), with zero response (for zero doppler offset) in the maximum unambiguous range interval, and in the third-time-around range interval. This radar waveform and processing could be useful for detecting targets in the second-time-around range interval while reducing clutter interference from clutter sources in the maximum unambiguous range interval.

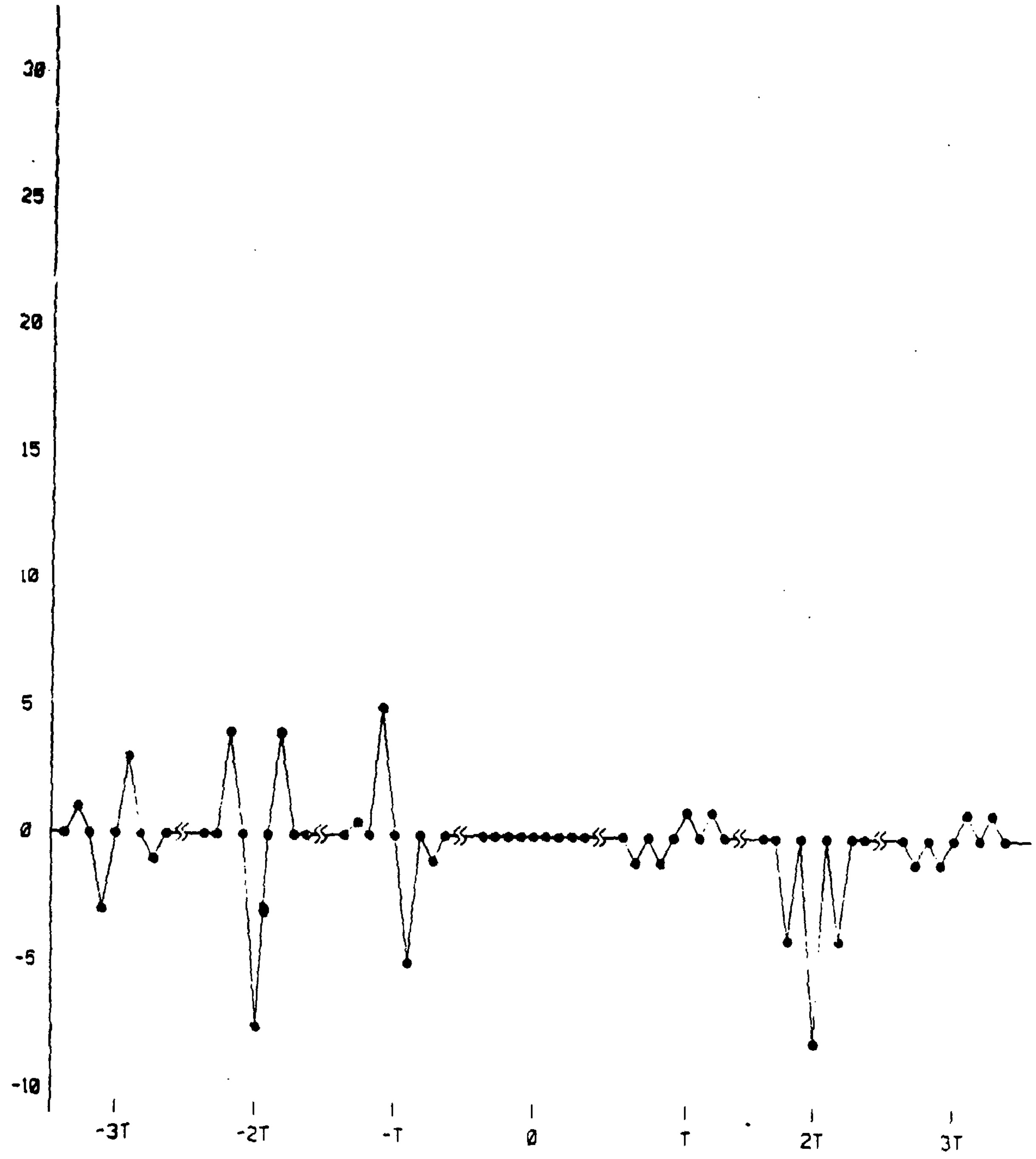
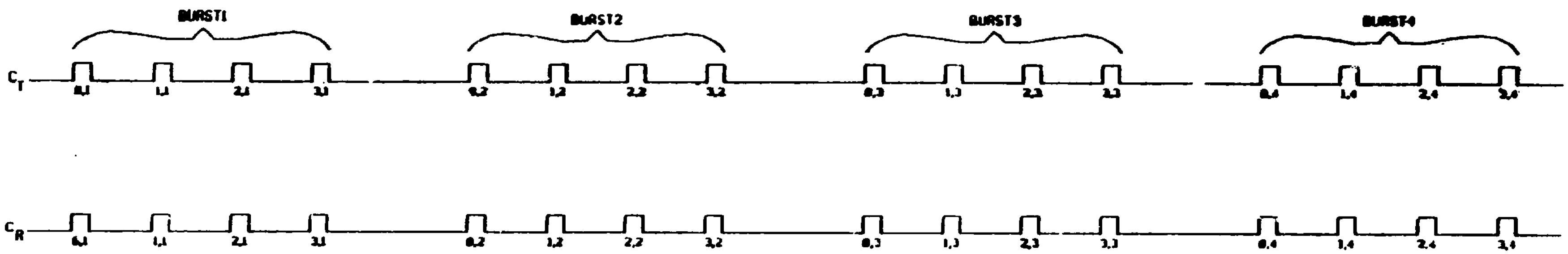


Figure 5. Cross-correlation between A0 and A3.



Note: Pulses labeled by array number and row number of bi-phase modulating

Figure 6. <u>Timing diagram for transmitted (C_T) and reference (C_R) coded pulse sequences.</u>

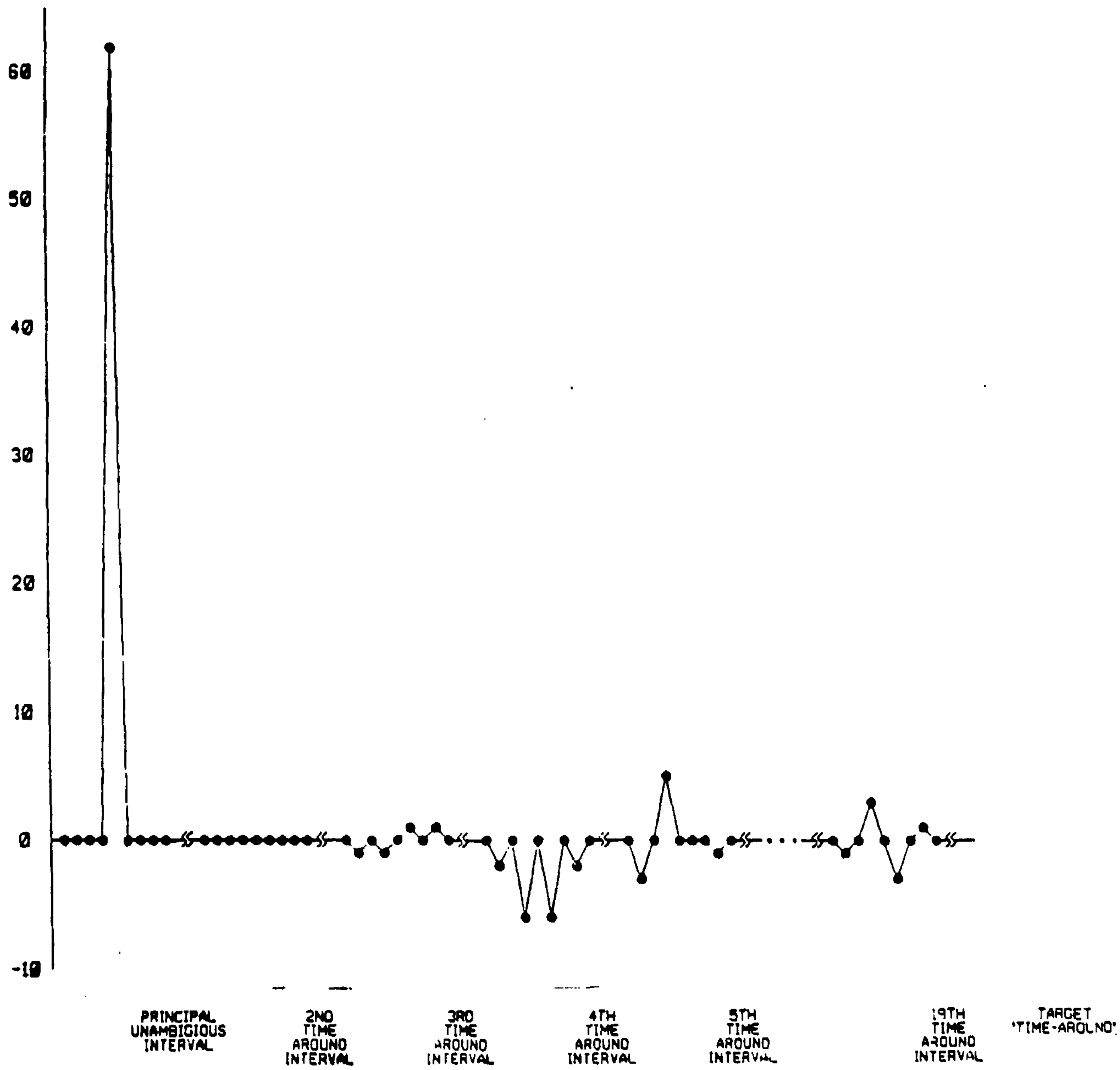


Figure 7. Cross-correlation between C_T and C_R.

For the example given, the time between each burst of four pulses is 2T. If this period is increased to 3T, then sidelobe cancellation is extended to the third-time-around range interval for the equivalent uncoded pulse sequence, and so on.

V. CONCLUSIONS

Sets of group-complementary code arrays which have the property of being mutually orthogonal over a subinterval of their total cross-correlation function have been identified. These group-orthogonal arrays are generated by a multiplication operation upon a given group-complementary array using a matrix which has diagonal elements from vectors which are orthogonal. Walsh vectors are examples of such orthogonal vectors. Composite code arrays can be synthesized by interleaving rows from the group-orthogonal set of arrays, and applying the resulting rows of codes to biphase modulate pulses in bursts. The resulting composite waveforms can have the desirable property of temporal (zero-doppler) sidelobe cancellation in the maximum unambiguous range interval and response cancellation over one or more multiple-time-around range intervals of an equivalent uncoded pulse sequence. The waveforms can also have the property of maximizing the receiver response in one of the multiple-time-around range intervals while achieving response cancellation in the maximum unambiguous range interval of an equivalent uncoded pulse sequence.

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